Method AiBi and Logistics for the projection of small areas: an application to micro-Angicos - RN

Cristiane Silva Corrêa – CEDEPLAR/UFMG e UFRN
Luana Junqueira Dias Myrrha – CEDEPLAR/UFMG e UFRN
Moema Fígoli – CEDEPLAR/UFMG

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Goals

1 Discuss two methods of projection of small area: the logistic and AiBi

2 Illustrate the application of methods in the population of micro-Angicos / RN, as the larger area and the eight municipalities that make up the large area.
Projected population of smaller areas as a function of a larger area.
Método AiBi

\[ P(t) = \sum_{i=1}^{n} P_i(t) \]

\[ P_i(t) = a_i P(t) + b_i \]

\[ a_i = \frac{P_i(t_1) - P_i(t_0)}{P(t_1) - P(t_0)} \quad b_i = P_i(t_0) - a_i P(t_0) \]

\( a_i \): the coefficient of proportionality of the increased population of smaller area \( i \) relative to the increase of the population of the larger area;

\( b_i \): the linear coefficient of correction
The coefficient $a_i$ reports the percentage growth of the larger area which the growth of the smaller area was responsible, between two periods.

$$\sum_{i=1}^{n} a_i = 1$$
Method AiBi

- Each population of smaller area $i$, at time $t$, is a proportion of the population of the larger area, corrected by a correction factor $b_i$. 
Coefficient $b_i$

- The coefficient $b_i$ is a correction factor that adjusts the relative distribution of the populations of small areas at time $t$, to the distribution at the beginning of the projection.
- $b_i$ can be less than 0.

$$\sum_{i=1}^{n} b_i = 0$$
Limits for bi:

- Considering that $a_i$ can only assume values between 0 and 1, we can define what are the possible limits for $b_i$:

$$\lim_{a_i \to 0} b_i = \lim_{a_i \to 0} P_i(t) - a_i P(t) = P_i(t)$$

$$\lim_{a_i \to 1} b_i = \lim_{a_i \to 1} P_i(t) - a_i P(t) = P_i(t) - P(t)$$
Method Limitation

- It is not consistent when the population growth of the larger area, and the smaller areas, have opposite directions.
Solution to Method Limitation

- Divide the larger area into two subgroups
  - smaller areas growing population
  - smaller areas that decrease in population.

- Problem: result affected by the criterion chosen to divide the population of the larger area.
Another solution: estimate $\Phi_i$

\begin{align*}
  P_i(t) &= a_i P(t) + b_i \\
  \frac{P_i(t)}{P(t)} &= \frac{b_i + a_i P(t)}{P_T(t)} \\
  \phi_i &= a_i + b_i / P(t)
\end{align*}

- The relationship between $P_i(t)$ and $P(t)$ changes from linear to hyperbolic.
- No more modeled $P_i(t)$ but $\Phi_i$
- $0 < \Phi_i < 1$. 
Relação hiperbólica entre \( F_i(t) \) e \( P(t) \)
New interpretation of the coefficients - $b_i$

- The sign of $b_i$ indicates the **concavity** of the hyperbola which gives the relationship between $P_i(t)$ and $P(t)$:
  - $b_i > 0$, the hyperbola is concave and $\Phi_i(t)$ is related to $P(t)$ in decreasing order;
  - $b_i < 0$, the hyperbola is convex and the relationship between $\Phi_i(t)$ and $P(t)$ is positive.

- The possibility of having negative and positive relationships between population of small and large areas is the main advantage of modeling $\Phi_i(t)$ instead of modeling $P_i(t)$ as the method AiBi.
New interpretation of the coefficients - $a_i$

- $A_i$: the limit of $\Phi_i$ when $P(t)$ tends to infinity.

![Graph showing $F_i$ vs. $P(t)$ with labels $a=0.6$ and $b=100$]
**Limites para Φi**

- Analyzing

\[ \lim_{P(t) \to 0} \Phi_i \quad \text{e} \quad \lim_{P(t) \to \infty} \Phi_i \]

**Tabela 5 – Limites de Φi para cada combinação de valores de ai e bi.**

<table>
<thead>
<tr>
<th>ai</th>
<th>bi</th>
<th>Limite Inferior (L₁)</th>
<th>Limite Superior (L₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>0</td>
<td>ai</td>
</tr>
<tr>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>ai</td>
<td>1</td>
</tr>
<tr>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Logistics function

The logistic function has the advantage of establishing a limit to growth

\[
\Phi_i(t) = L_1 + \frac{L_2 - L_1}{1 + \exp \{\alpha + \beta(t_1 - t_0)\}}
\]

\[
\beta = \frac{1}{t_1 - t_0} \left( \ln \left( \frac{L_2 - \Phi_i(t_1)}{\Phi_i(t_1) - L_1} \right) - \alpha \right)
\]

\[
\alpha = \ln \left( \frac{L_2 - \Phi_i(0)}{\Phi_i(0) - L_1} \right)
\]
Application of the Method

- Micro Angicos - RN
- 8 municipalities: Afonso Bezerra, Angicos, Wind River Caiçara Fernando Pedroza, Garden Angicos, Plates, Black Stone, Pedro Avelino.
- Total population in 2010 estimated by the exponential growth rate between 1991 and 2000.
Tabela 7 - Observed population and the population estimated by Ai Bi and Logistics method, by municipality, Angicos - RN, 2010

<table>
<thead>
<tr>
<th>Município</th>
<th>População Recenseada 2010</th>
<th>População Estimada AiBi</th>
<th>População Estimada Logístico</th>
</tr>
</thead>
<tbody>
<tr>
<td>Afonso Bezerra</td>
<td>10844</td>
<td>11022</td>
<td>10457</td>
</tr>
<tr>
<td>Angicos</td>
<td>11549</td>
<td>11731</td>
<td>11203</td>
</tr>
<tr>
<td>Caiçara do Rio do Vento</td>
<td>3308</td>
<td>3158</td>
<td>2694</td>
</tr>
<tr>
<td>Fernando Pedroza</td>
<td>2854</td>
<td>2414</td>
<td>2641</td>
</tr>
<tr>
<td>Jardim de Angicos</td>
<td>2607</td>
<td>2938</td>
<td>2510</td>
</tr>
<tr>
<td>Lajes</td>
<td>10381</td>
<td>10224</td>
<td>8868</td>
</tr>
<tr>
<td>Pedra Preta</td>
<td>2590</td>
<td>3006</td>
<td>2709</td>
</tr>
<tr>
<td>Pedro Avelino</td>
<td>7171</td>
<td>5219</td>
<td>8628</td>
</tr>
<tr>
<td>Total</td>
<td>51304</td>
<td>49711</td>
<td>49711</td>
</tr>
</tbody>
</table>

Conclusion

- For the micro-Angicos - RN, the logistic method yielded better results.

<table>
<thead>
<tr>
<th>Método</th>
<th>Soma dos Quadrados dos Erros</th>
</tr>
</thead>
<tbody>
<tr>
<td>AiBi</td>
<td>4.398.982</td>
</tr>
<tr>
<td>Logístico</td>
<td>2.335.101</td>
</tr>
</tbody>
</table>
Referências:

Cristiane Silva Corrêa
criscorrea@ufmg.br